Closing Tue: 12.5(1)(2), 12.6
Closing Thu: 10.1/13.1
Closing next Tue: 10.2/13.2, 10.3

12.6: A few (*seven*) basic 3D surfaces

First, a 2D review.

Line: $ax + by = c$

Parabola: $ax^2 + by = c$ or
$ax + by^2 = c$

Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

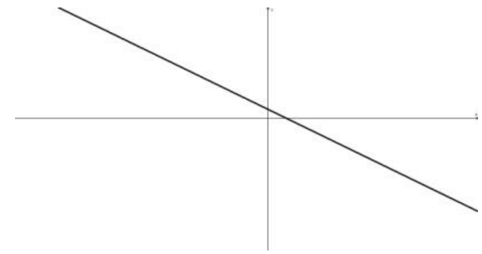
(*Note:* If a = b, it's a circle)

Hyperbola: $ax^2 - by^2 = c$ or
$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

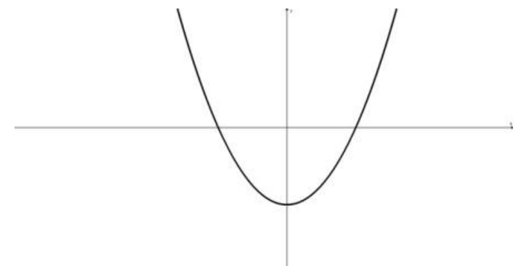
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Examples:

$3x + 2y = 1$
(Line)

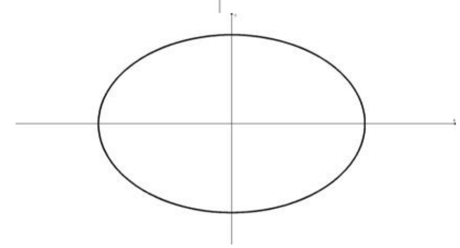


$3x^2 - y = 4$
(Parabola)



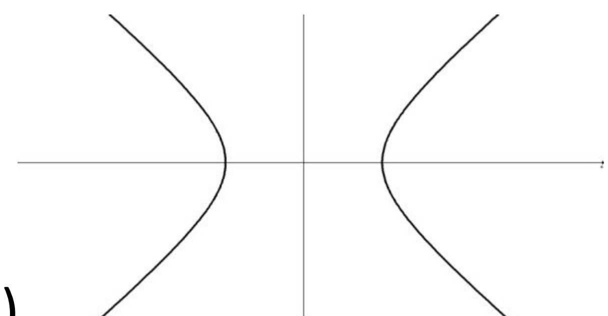
$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

(Ellipse)



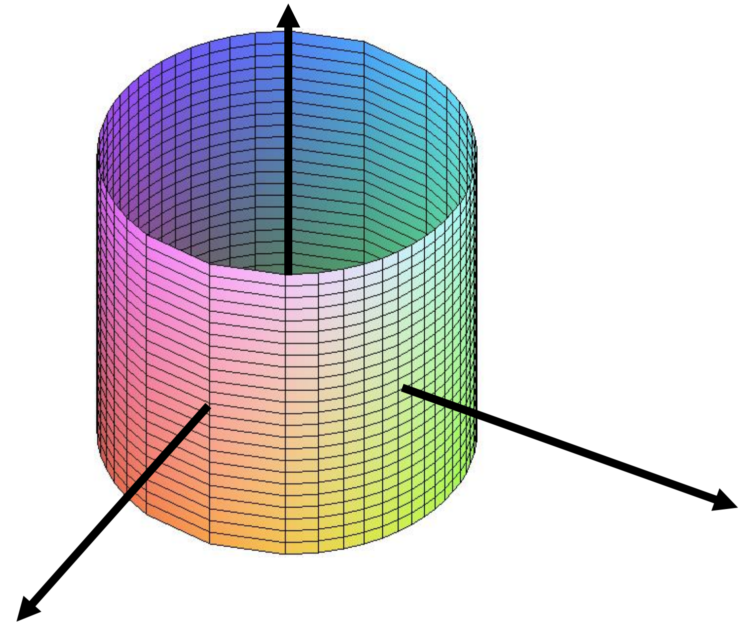
$$\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$$

(Hyperbola)



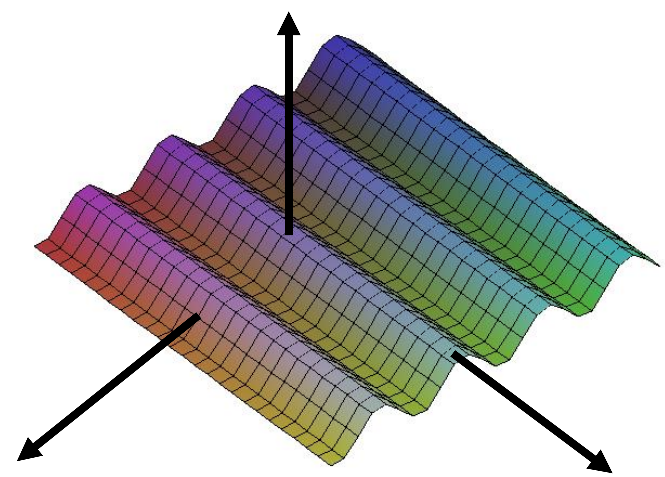
Shape 1: Cylinders

If *one variable is absent*, then the graph is a 2D curve extended into 3D. If the 2D shade is called "BLAH", then the 3D shade is a "BLAH cylinder".



Examples:

(a) $x^2 + y^2 = 1$ in 3D is a **circular cylinder** (*i.e.* a circle extended in the *z*-axis direction).



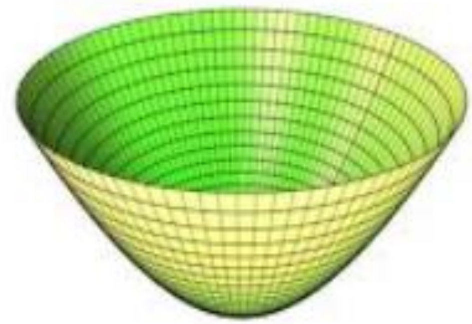
(b) $z = \cos(x)$ in 3D is a **cosine cylinder** (i.e. the cosine function extended in the *y*-axis direction).

Quadric Surfaces: A surface given by an equation involving a sum of first and second powers of *x*, *y*, and *z* is called a *quadric surface*.

To visualize, we use **traces**:
Fix one variable and look at the resulting 2D picture (i.e. look at one slice).

If we do several traces in different directions, we start to get an idea about the picture.

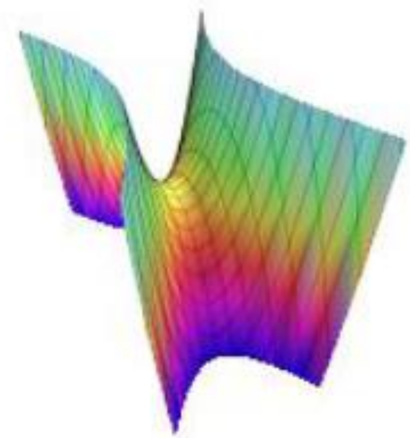
Shapes 2 and 3: Paraboloids



Elliptical/Circular Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

(*ex*: $z = 3x^2 + 5y^2$)



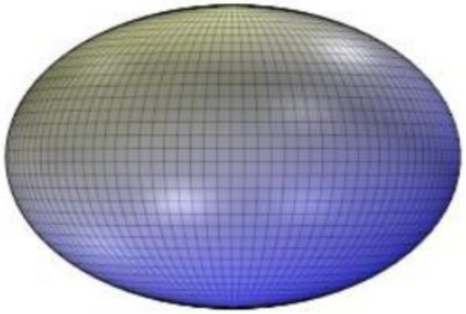
Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

(*ex*: $y = 2x^2 - 5z^2$)

Shapes 4: Ellipsoid

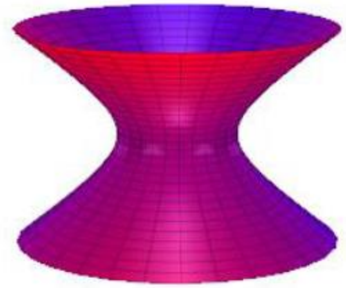
Shapes 5, 6, 7: 3 types of hyperboloids



Ellipsoid/Sphere

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

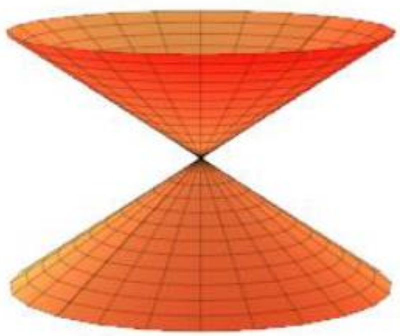
(*ex*: $3x^2 + 5y^2 + z^2 = 3$)



Hyperboloid of One Sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

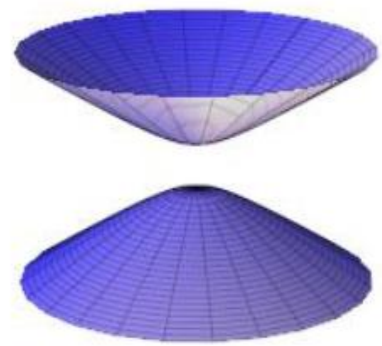
(*ex*: $x^2 - y^2 + z^2 = 10$)



Circular/Elliptical Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

(*ex*: $z^2 = x^2 + y^2$)



Hyperboloid of Two Sheets

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

(*ex*: $x^2 + y^2 - z^2 = -4$)

Practice Examples

Find the traces and name the shapes:

1. $x - 3y^2 + 2z^2 = 0$
2. $4x^2+3y^2 = 10$
3. $5x^2 - y^2 - z^2 = 4$
4. $-x^2 + y^2 + 4z^2 = 0$
5. $x^2 - 2y^2 + z^2 - 6 = 0$

Answers:

1. $x - 3y^2 + 2z^2 = 0$

$x = k$: $k - 3y^2 + 2z^2 = 0$ (**hyp.**)

$y = k$: $x - 3k^2 + 2z^2 = 0$ (**par.**)

$z = k$: $x - 3y^2 + 2k^2 = 0$ (**par.**)

Also note: $x = 3y^2 - 2z^2$

Name: **Hyperbolic paraboloid**

2. $4x^2+3y^2 = 10$

One variable missing.
The given equation is an ellipse in the xy-plane.

Name: **Elliptical Cylinder**

3. $5x^2 – y^2 – z^2 = 4$

$x = k$: $5k^2 – y^2 – z^2 = 4$ (circ/pt/nothing)
$y = k$: $5x^2 – k^2 – z^2 = 4$ (**hyp**)
$z = k$: $5x^2 – y^2 – k^2 = 4$ (**hyp**)
Also note: $-5x^2 + y^2 + z^2 = -4$

Name: **Hyperboloid of Two Sheets**

4. $–x^2 + y^2 + 4z^2 = 0$

$x = k$: $-k^2 + y^2 + 4z^2 = 0$ (**ellipse/pt**)
$y = k$: $- x^2 + k^2 + 4z^2 = 0$ (**hyp./lines**)
$z = k$: $- x^2 + y^2 + 4k^2 = 0$ (**hyp./lines**)
Also note: $x^2 = y^2 + 4z^2$

Name: **Elliptical Cone**

5. $x^2 - 2y^2 + z^2 – 6 = 0$

$x = k$: $k^2 – 2y^2 + z^2 – 6 = 0$ (**hyp**)
$y = k$: $x^2 – 2k^2 + z^2 – 6 = 0$ (**circle**)
$z = k$: $x^2 – 2y^2 + k^2 – 6 = 0$ (**hyp**)

Also note: $x^2 – 2y^2 + z^2 = 6$

Name: **Hyperboloid of One Sheet**